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APPROXIMATION OF FUNCTION ASSOCIATED WITH HARDY-LITTLEWOOD SERIES IN THE HÖLDER METRIC BY MEAN

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Abstract— In the present paper we have studied the degree of approximation of a function associated with Hardy-Littlewood series using $[S,\alpha_n]$ mean. Our result is analogous to the result obtained by U.K.Srivastava and C.J.Rathore [5].

Index Terms— Banach space, Hardy-Littlewood series, Holder metric ,Fourier series.

1 Introduction

1. **Definition**

Let $C_{2\pi}$ denotes the Banach space of all periodic continuous functions defined on $[-\pi,\pi]$ under sup norm. For $0<\alpha\leq 1$ and positive constant K. The function space is given by

$$H_\alpha = \left\{ f \in C_{2\pi} : \left| f(x) - f(y) \right| \leq K \left| x-y \right|^\alpha \right\} \tag{1.1}$$

The space H_α is a Banach space with the norm $\| \ \|_\alpha$ defined by

$$\|f\|_\alpha = \|f\|_c = \sup_{x,y} \left\{ \Delta^\alpha f(x,y) \right\} \tag{1.2}$$

where $\|f\|_c = \sup\limits_{-\pi\leq x\leq\pi} |f(x)|$ *and*

$$\Delta^\alpha f(x,y) = \left| f(x) - f(y) \right| \left| x-y \right|^{-\alpha} \qquad x \neq y \tag{1.3}$$

We shall use the convention that

$$\Delta^\alpha f(x,y) = 0.$$

The metric induced by norm (1.2) on H_α is called Hölder metric.

Let f be a periodic function of period 2π and integrable in the Lebesgue sense over $[-\pi,\pi]$.Let the Fourier series associated with f at x be

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} A_n(x) \tag{1.4}$$

Let us write

$$\phi_x(u) = \frac{1}{2} \left\{ f(x+u) + f(x-u) - 2f(x) \right\} \tag{1.5}$$

$$\chi_x(u) = \int_u^\pi \phi_x(w) \frac{1}{2} \cot \frac{w}{2} \, dw \tag{1.6}$$

Let $S_n(x)$ and $S_n^*(x)$ respectively denote the partial sum and modified partial sum of (1.4).

i.e $S_n(x) = \sum\limits_{k=0}^{n} A_k(x)$, $S_n^*(x) = \sum\limits_{k=0}^{n-1} A_k(x) + \frac{1}{2} A_n(x)$

It is known [6] that

$$S_n^*(x) - f(x) = \frac{2}{\pi} \int_0^\pi \frac{\phi_x(u) \sin nu}{2 \tan \frac{u}{2}} du \tag{1.7}$$

For the first time Meir and Sharma [3] introduced generalization of the S_α method and called it $[S,\alpha_n]$ method. They obtained sufficient condition for the regularity of this method. They also examined the behavior of its Lebesgue constant.

Let $\{\alpha_j\}$ be a given sequence of real complex numbers. We shall say that $\{\alpha_j\}$ is the $[S, \alpha_n]$ transformations of $\{S_j\}$; i.e. the sequence of partial sum of the series $\sum \alpha_n$ if

$$\{\sigma_n\} = \sum_{k=0}^{\infty} C_{nk} S_k \text{ ; (n=0, 1, 2, 3,…)} \tag{1.8}$$

Converges, where (C_{nk}) is given by the identity

$$\prod \frac{1-\alpha_j}{1-\alpha_j \theta} = \sum_{k=0}^{\infty} C_{nk} \theta^k \tag{1.9}$$

The sequence $\{S_j\}$ is said to be $[S, \alpha_n]$ summable to σ if

$$\lim_{n\to\infty} \sigma_n = \sigma$$

Let f(x) ∈L (0, 2π) and be periodic with period 2π outside this range. Let the Fourier series associated with the function be

$$\frac{a_0}{2} + \sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx) = \sum_{n=0}^{\infty} A_n(x) \tag{1.10}$$

and as usual we denote

$$\phi(t) = \phi_x(t) = \frac{1}{2}\{f(x+t) + f(x-t) - 2f(x)\} \tag{1.11}$$

Also

$$V_n = 1 + 2\sum_{j=0}^{n} \frac{\alpha_j}{1-\alpha_j} \tag{1.12}$$

$$T_n = 2\sum_{j=0}^{n} \frac{\alpha_j}{(1-\alpha_j)^2} \tag{1.13}$$

$$[T_n], \text{ the integral part of } T_n \tag{1.14}$$

2. Introduction

$$\sum_{n=1}^{\infty} \frac{S_n(x) - f(x)}{n} \tag{2.1}$$

denotes Hardy-Littlewood series or in short HL- series.

Hardy and Littlewood([2]) have shown that (2.1) is summable (C,1) to the value

$$\frac{1}{\pi}\int_0^{\pi} \left\{ \left(\frac{\pi-u}{2}\right)\cot\frac{u}{2} - \log\left(2\sin\frac{u}{2}\right) \right\} \phi_x(u)\, du$$

whenever integral

$$\int_{0^+}^{\pi} \phi_x(u) \frac{1}{2}\cot\frac{u}{2}\, du \tag{2.2}$$

exists. Further from [2] and [6] if

$$\int_0^t |\phi(u)|\, du = o(t) \qquad \text{as } t \to 0^+ \tag{2.3}$$

Then (2.1) converges if and only if (2.2) exists. The relation of HL- series with the integral (2.2) similar to those between the conjugate series $\sum B_x(x)$ and the integral

$$\int_{0^+}^{\pi} \frac{\psi_x(u)}{u}\, du \tag{2.4}$$

where $\psi_x(u) = \frac{1}{2}\{f(x+u) - f(x-u)\}$.

It is known that if $f \in L[-\pi, \pi]$ then (2.4) exists almost everywhere. On the other hand there exists a continuous function f for which the integral (2.2) diverges for almost all x [1]. At this stage we remark that the above results on HL- series remain unaltered if we replace the HL-series by

$$\frac{C_0}{2} + \sum_{n=1}^{\infty} \frac{S_n^*(x) - f(x)}{n} \tag{2.5}$$

where $C_0 = \frac{2}{\pi}\int_0^{\pi} \phi_x(u) \frac{u}{2}\cot\frac{u}{2}\, du$

The series (2.5) is summable (C, 1) to the value

$$\int_0^{\pi} \phi_x(u) \frac{1}{2}\cot\frac{u}{2}\, du$$

whenever this integral exists. Thus convergence or summability problem of (2.5) is same as HL-series.

Let $T_n(x)$ denote the n-th partial sum of the series (2.5)

$$\text{i.e } T_n(x) = \frac{C_0}{2} + \sum_{k=1}^{n} \frac{S_n^*(u) - f(x)}{k} \tag{2.6}$$

3. Theorems

Das ([1]) has studied the degree of approximation of the series (2.5) by Euler's mean and proved the following theorem.

Theorem1- If $0\leq\beta<\alpha\leq 1$ and $f \in H_\alpha$ then

$$\left\|E_n^q(T)-\chi(N)\right\|_\beta = O\left(n^{\beta-\alpha}(\log n)^{1+\frac{\beta}{\alpha}}\right) \tag{3.1}$$

where $N=\dfrac{\pi(1+q)}{n}$ and $E_n^q(T,x)$ is the Euler's transformation of $T_n(x)$.

U.K.Shrivastava and C.J.Rathore[5] extend the above result by proving following approximation result for [F, dn] transform. The theorem is as follows

Theorem 2- If $0\leq\beta<\alpha\leq 1$ and $f \in H_\alpha$ then

$$\left\|\sigma_n(f)-f\right\|_\beta = 0\left(n^{\beta-\alpha}\log n\right).$$

where $\sigma_n(f)$ is $[F,dn]$ transformation of $T_n(x)$.

2 Procedure for Paper Submission

2.1 Review Stage

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Abstract	In the present paper we have studied the degree of approximation of a function associated with Hardy-Littlewood series using mean. Our result is analogous to the result obtained by U.K.Srivastava and C.J.Rathore [5].
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$$\int_0^{r_2} F(r,\varphi)\, dr\, d\varphi = [\sigma r_2 / (2\mu_0)] \cdot \int_0^{\infty} \exp(-\lambda |z_j - z_i|)\, \lambda^{-1} J_1(\lambda r_2)\, J_0(\lambda r_i)\, d\lambda . \tag{1}$$

Be sure that the symbols in your equation have been defined before the equation appears or immediately following. Italicize symbols (T might refer to temperature, but T is the unit tesla). Per IJSER, please refer to "(1)," not "Eq. (1)" or "equation (1)," except at the beginning of a sentence: "Equation (1) shows ... ." Also see *The Handbook of Writing for the Mathematical Sciences*, 1993. Published by the Society for Industrial and Applied Mathematics, this handbook provides some helpful information about math typography and other stylistic matters. For further information about typesetting mathematical equations, please visit the IJSER styel guide: http://www.ijser.org.

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TABLE 1
UNITS FOR MAGENTIC PROPERTIES

Symbol	Quantity	Conversion from Gaussian and CGS EMU to SI [a]
Φ	magnetic flux	1 Mx $\rightarrow 10^{-8}$ Wb = 10^{-8} V·s
B	magnetic flux density, magnetic induction	1 G $\rightarrow 10^{-4}$ T = 10^{-4} Wb/m^2
H	magnetic field strength	1 Oe $\rightarrow 10^3/(4\pi)$ A/m
m	magnetic moment	1 erg/G = 1 emu $\rightarrow 10^{-3}$ A·m^2 = 10^{-3} J/T
M	magnetization	1 erg/(G·cm^3) = 1 emu/cm^3 $\rightarrow 10^3$ A/m
$4\pi M$	magnetization	1 G $\rightarrow 10^3/(4\pi)$ A/m
σ	specific magnetization	1 erg/(G·g) = 1 emu/g $\rightarrow$ 1 A·m^2/kg
j	magnetic dipole moment	1 erg/G = 1 emu $\rightarrow 4\pi \times 10^{-10}$ Wb·m
J	magnetic polarization	1 erg/(G·cm^3) = 1 emu/cm^3 $\rightarrow 4\pi \times 10^{-4}$ T
χ, κ	susceptibility	$1 \rightarrow 4\pi$
χ_ρ	mass susceptibility	1 cm^3/g $\rightarrow 4\pi \times 10^{-3}$ m^3/kg
μ	permeability	$1 \rightarrow 4\pi \times 10^{-7}$ H/m = $4\pi \times 10^{-7}$ Wb/(A·m)
μ_r	relative permeability	$\mu \rightarrow \mu_r$
w, W	energy density	1 erg/cm^3 $\rightarrow 10^{-1}$ J/m^3
N, D	demagnetizing factor	$1 \rightarrow 1/(4\pi)$

Statements that serve as captions for the entire table do not need footnote letters.

[a]Gaussian units are the same as cgs emu for magnetostatics; Mx = maxwell, G = gauss, Oe = oersted; Wb = weber, V = volt, s = second, T = tesla, m = meter, A = ampere, J = joule, kg = kilogram, H = henry.

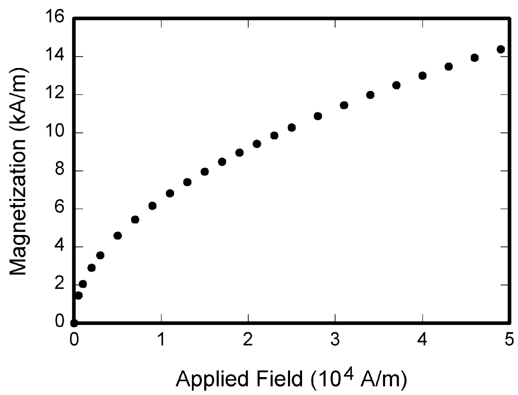

Fig. 1. Magnetization as a function of applied field. Note that "Fig." is abbreviated. There is a period after the figure number, followed by one space. It is good practice to briefly explain the significance of the figure in the caption.

Figure axis labels are often a source of confusion. Use words rather than symbols. As an example, write the quantity "Magnetization," or "Magnetization *M*," not just "*M*." Put units in parentheses. Do not label axes only with units. As in Fig. 1, for example, write "Magnetization (A/m)" or "Magnetization (A $\cdot$ m^{-1})," not just "A/m." Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)," not "Temperature/K." Table 1 shows some examples of units of measure.

Multipliers can be especially confusing. Write "Magnetization (kA/m)" or "Magnetization (103 A/m)." Do not write "Magnetization (A/m) × 1,000" because the reader would not know whether the top axis label in Fig. 1 meant 16,000 A/m or 0.016 A/m. Figure labels should be legible, approximately 8 to 12 point type. When creating your graphics, especially in complex graphs and charts, please ensure that line weights are thick enough that when reproduced at print size, they will still be legible. We suggest at least 1 point.

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Proof. The same format should be used for structures such as remarks, examples, and solutions (though these would not have a Q.E.D. box at the end as a proof does).

7 END SECTIONS

7.1 Appendices

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Acknowledgment

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